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UNSYMMETRICAL CONCRETE ARCHES—DETERMINATION OF MOMENT, SHEAR AND THRUST AT CROWN

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(Presented at a meeting of the Designers Section of the Boston Society of Civil Engineers,
March 8, 1933)

SYNOPSIS

THE authors of many of our textbooks on concrete structures limit their treatment of the design of concrete arches to the symmetrical case of the single span right arch, although they often give directions whereby the stresses in unsymmetrical arches may be computed. Technical papers, on the other hand, are apt to plunge into discussions rather remote from the simple case. A transitional paper, dealing with one method of treatment of the unsymmetrical single right fixed arch, can be very profitable, especially if it confines the discussion to familiar terms and relations.

This paper was written for the members of the Designers Section of the Boston Society of Civil Engineers, and was designed to provide an hour's discussion of an interesting topic rather than to advance engineering knowledge in this field. It assumes that one is familiar with the standard methods of computing the moment, shear and thrust in the case of the symmetrical arch. It reviews the derivation of the basic equations for the deflection of curved beams, and derives a method, based upon simultaneous equations, for the determination of the moment, shear and thrust at the crown of an unsymmetrical right arch. When these quantities have been found, the designer can readily determine the desired quantities at any other point in the arch by means of statics. A method of solving for the horizontal thrust at the hinges of an unsymmetrical two-hinged arch is then presented.

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The illustrative problem is taken, with the permission of the author, from Prof. Hardy Cross's paper, "The Column Analogy," Bulletin No. 215, Engineering Experiment Station, University of Illinois, which may be consulted for an excellent method of solving this and other more difficult problems.

Note. — This paper does not discuss questions of actual design; it is limited to the determination of certain quantities in a certain frame. Nor is there a discussion of the validity of the elastic theory. (See Swain, "Stresses, Graphical Statics and Masonry," page 425; Hardy Cross, "Dependability of the Theory of Concrete Arches," Bulletin No. 203, Engineering Experiment Station, University of Illinois; Discussion by Edward Godfrey, Transactions, American Society of Civil Engineers, Vol. 88, 1925, page 1189.) Neither is plastic flow discussed, although this question is becoming of absorbing interest to engineers. (See "Plain and Reinforced Concrete Arches," Journal of the American Concrete Institute, March, 1932, and "Principles of Reinforced Concrete Construction," Turneaure and Maurer, fourth edition, page 349; also bulletins of the American Society for Testing Materials.) The writer assumes, contrary to fact, that abutments are immovable, that E and I are known, and that a plane section before bending remains a plane section after bending, thus neglecting any slight deformation due to shear; that is, one must still use his engineering judgment in the use of the equations herein presented.

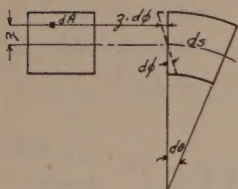
NOTATION (FIXED ARCH)

- | | |
|--|---|
| H_o = thrust at the origin (elastic center) | } Positive when acting, as shown in Fig. 4. |
| M_o = moment at the origin. | |
| V_o = shear at the origin | |
| I = moment of inertia of any section. | |
| E = modulus of elasticity. | |
| Δs = length of any segment measured along the arch axis. | |
| x and y are co-ordinates of any point on the arch axis measured from the origin at the elastic center, positive when acting as shown in Fig. 4. | |
| m = moment about the center of any segment due to applied loads and weight of arch ring acting between the point under consideration and the cut section at the crown. | |
| M = the total moment about the center of any segment due to all forces acting between it and the cut section at the crown, positive when causing compression in the upper fibers of the rib. (See Fig. 4.) | |
| The subscripts R and L indicate that the segment under consideration is to the right or left, respectively, of the crown. | |
| X and Y are axes passing through the elastic center of the arch. | |
| $\Delta\phi$ = change in angle of the tangent to the arch axis, positive when acting as shown in Fig. 3. | |

The elastic center, or centroid, is the point about which $\int \frac{y \cdot ds}{EI}$ is zero and $\int \frac{x \cdot ds}{EI}$ is zero; that is, considering the quantities $\frac{\Delta s}{I}$ as weights, the elastic center is the center of gravity of these weights. (See page 102.)

The terms peculiar to the two-hinged arch are mentioned in the text.

STRESSES IN CURVED BEAMS (A REMINDER)



$E = \frac{\text{stress}}{\text{strain}}$ i.e., Stress on $dA = E \cdot z \cdot \frac{d\phi}{ds}$

Force on element $dA = E \cdot z \cdot \frac{d\phi}{ds} \cdot dA$

Moment over section $= \int E \cdot z \cdot \frac{d\phi}{ds} \cdot z \cdot dA$

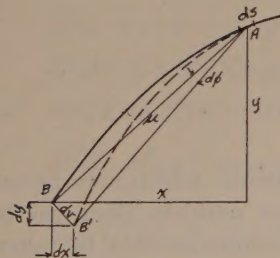
$M = \frac{E \cdot d\phi}{ds} \int z^2 \cdot dA = EI \frac{d\phi}{ds}$

i.e., $d\phi = \frac{M \cdot ds}{EI}$ = change in angle between end faces due to moment.

$\Delta \phi = \int_A^B \frac{M \cdot ds}{EI} \quad (1)$

FIG. 1

DEFLECTION OF CURVED BEAMS



In Fig. 2, let AB be the unstrained position of the arch axis, and AB' its position due to the distortion of the segment ds .

From similar triangles, noting that $dv = u \cdot d\phi$,

$\frac{dy}{dv} = \frac{x}{u}$; $\frac{dy}{u \cdot d\phi} = \frac{x}{u}$; $dy = x \cdot d\phi$

$\frac{dx}{dv} = \frac{y}{u}$; $\frac{dx}{u \cdot d\phi} = \frac{y}{u}$; $dx = y \cdot d\phi$

From (1)

$dx = \frac{M \cdot y \cdot ds}{EI}$, and $\Delta x = \int_A^B \frac{M \cdot y}{EI} \cdot ds \quad (2)$

$dy = \frac{M \cdot x \cdot ds}{EI}$, and $\Delta y = \int_A^B \frac{M \cdot x}{EI} \cdot ds \quad (3)$

FIG. 2

THE HINGELESS ARCH WITH FIXED ENDS

Equations of Displacement

Fig. 3 (a) shows the strained and unstrained axis of a fixed ended arch. Under load the point A , near the crown, has moved to the point A' . Fig. 3 (b) shows the arch cut at the crown, and indicates

References. — For a more complete exposition of the above, see Johnson, Bryan and Turneaure, "Modern Framed Structures," Vol. II, pp. 112-117; Williams, "Masonry Structures," pp. 160-162; Sutherland and Clifford, "Reinforced Concrete Design;" Turneaure and Maurer, "Principles of Reinforced Concrete Construction;" Spofford, "Theory of Structures;" Cross and Morgan, "Continuous Frames of Reinforced Concrete."

positive directions and rotations. It will be noted that a horizontal displacement of the cut end which is positive for one side will be negative for the other; that a rotation which is positive for one side will be negative for the other; and that a vertical displacement which is positive for one side will be positive for the other. Hence, as the total change in lateral, vertical and rotational displacements between abutments must be zero —

$$\left. \begin{aligned} \Delta \phi &= 0 \\ \Delta X &= 0 \\ \Delta Y &= 0 \end{aligned} \right\} (4) \text{ or, } \left. \begin{array}{ll} \text{L.H. Side} & \text{R.H. Side} \\ \Delta \phi_L &= -\Delta \phi_R \\ \Delta X_L &= -\Delta X_R \\ \Delta Y_L &= \Delta Y_R \end{array} \right\} (5)$$

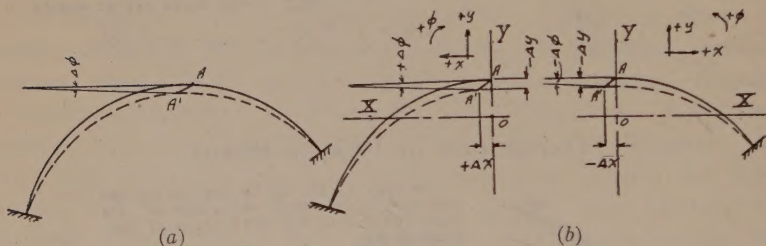


FIG. 3

Procedure

The arch is divided into a number of segments, which may be taken conveniently as uniform along the arch axis, or uniform along the horizontal axis, as in this paper. The origin of co-ordinates may be taken at either end, at the crown, or at the elastic center, as in this paper. In any case, the displacement of the cut end due to the distortion of each segment is found, and the results added to give the total effect, an approximate method of integration.

Fig. 4 shows the arch axis, with co-ordinates and indicated loads. The elastic center may be found from —

$$f = \frac{\sum \frac{ds^3}{EI}}{\sum \frac{ds}{EI}} ; \quad g = \frac{\sum X' \frac{ds}{EI}}{\sum \frac{ds}{EI}} \quad (\text{See "Notation"})$$

$$\text{i.e., } \sum_A^C Y \frac{ds}{EI} = 0 ; \quad \sum_A^C X \frac{ds}{EI} - \sum_B^C X \frac{ds}{EI} = 0 .$$

E is assumed to be constant, and may be eliminated from the equations. If ds/I is made constant, this term may be eliminated also.

As M is the total moment about any segment, by statics —

$$M_L = M_0 - H_0 \cdot y + V_0 \cdot x - m_L, \text{ and } M_R = M_0 - H_0 \cdot y - V_0 \cdot x - m_R.$$

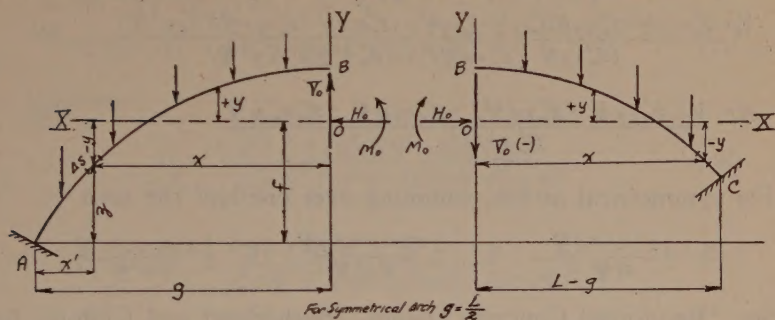


FIG. 4

Derivation of Equations

$$\text{From (5) and (1), } \Delta \phi_L + \Delta \phi_R = 0 \therefore \sum_n^B M_L \frac{\Delta \phi}{\Delta x} + \sum_n^C M_R \frac{\Delta \phi}{\Delta x} = 0$$

$$\sum_n^B (M_0 - H_0 \cdot y + V_0 \cdot x - m_L) \frac{\Delta \phi}{\Delta x} + \sum_n^C (M_0 - H_0 \cdot y - V_0 \cdot x - m_R) \frac{\Delta \phi}{\Delta x} = 0$$

$$\sum_n^B M_0 \frac{\Delta \phi}{\Delta x} + \sum_n^C M_0 \frac{\Delta \phi}{\Delta x} - \sum_n^B H_0 \cdot y \frac{\Delta \phi}{\Delta x} - \sum_n^C H_0 \cdot y \frac{\Delta \phi}{\Delta x} + \sum_n^B V_0 \cdot x \frac{\Delta \phi}{\Delta x} - \sum_n^C V_0 \cdot x \frac{\Delta \phi}{\Delta x} - \sum_n^B m_L \frac{\Delta \phi}{\Delta x} - \sum_n^C m_R \frac{\Delta \phi}{\Delta x} = 0$$

$$M_0 \sum_n^B \frac{\Delta \phi}{\Delta x} - H_0 \sum_n^B y \frac{\Delta \phi}{\Delta x} + V_0 (\sum_n^B x \frac{\Delta \phi}{\Delta x} - \sum_n^C x \frac{\Delta \phi}{\Delta x}) - \sum_n^B m_L \frac{\Delta \phi}{\Delta x} - \sum_n^C m_R \frac{\Delta \phi}{\Delta x} = 0$$

$$\text{As the 2^d and 3^d terms are zero, } M_0 = \frac{\sum_n^B m_L \frac{\Delta \phi}{\Delta x} + \sum_n^C m_R \frac{\Delta \phi}{\Delta x}}{\sum_n^B \frac{\Delta \phi}{\Delta x}} \quad (6)$$

$$\text{From (5) and (2), } \Delta x_L + \Delta x_R = 0 \therefore \sum_n^B M_L y \frac{\Delta \phi}{\Delta x} + \sum_n^C M_R y \frac{\Delta \phi}{\Delta x} = 0$$

$$\sum_n^B (M_0 - H_0 \cdot y + V_0 \cdot x - m_L) y \frac{\Delta \phi}{\Delta x} + \sum_n^C (M_0 - H_0 \cdot y - V_0 \cdot x - m_R) y \frac{\Delta \phi}{\Delta x} = 0$$

$$\text{Combining terms, and noting that } M_0 \sum_n^C y \frac{\Delta \phi}{\Delta x} = 0,$$

$$-H_0 \sum_n^C y^2 \frac{\Delta \phi}{\Delta x} + V_0 (\sum_n^B x y \frac{\Delta \phi}{\Delta x} - \sum_n^C x y \frac{\Delta \phi}{\Delta x}) - \sum_n^B m_L y \frac{\Delta \phi}{\Delta x} - \sum_n^C m_R y \frac{\Delta \phi}{\Delta x} = 0 \quad (7)$$

$$\text{From (5) and (3), } \Delta y_L - \Delta y_R = 0 \therefore \sum_n^B M_L x \frac{\Delta \phi}{\Delta x} - \sum_n^C M_R \cdot x \frac{\Delta \phi}{\Delta x} = 0$$

$$\sum_n^B (M_0 - H_0 \cdot y + V_0 \cdot x - m_L) x \frac{\Delta \phi}{\Delta x} - \sum_n^C (M_0 - H_0 \cdot y - V_0 \cdot x - m_R) x \frac{\Delta \phi}{\Delta x} = 0$$

$$\text{Combining terms, and noting that } M_0 (\sum_n^B x \frac{\Delta \phi}{\Delta x} - \sum_n^C x \frac{\Delta \phi}{\Delta x}) = 0,$$

$$-H_0 (\sum_n^B x y \frac{\Delta \phi}{\Delta x} - \sum_n^C x y \frac{\Delta \phi}{\Delta x}) + V_0 \sum_n^B x^2 \frac{\Delta \phi}{\Delta x} - \sum_n^B m_L \cdot x \frac{\Delta \phi}{\Delta x} + \sum_n^C m_R \cdot x \frac{\Delta \phi}{\Delta x} = 0. \quad (8)$$

M_0 may be found directly from (6). H_0 and V_0 may be found by substituting into (7) and (8) and solving the resulting equations

simultaneously; or these equations may be solved for H_o and V_o as follows:

$$V_o = \frac{(\sum_a^o m_L y \frac{A^3}{E I} + \sum_o^c m_R y \frac{A^3}{E I} (\sum_a^o x y \frac{A^3}{E I} - \sum_o^c x y \frac{A^3}{E I}) - (\sum_a^o m_L x \frac{A^3}{E I} - \sum_o^c m_R x \frac{A^3}{E I}) (\sum_a^c y^2 \frac{A^3}{E I})}{(\sum_a^o x y \frac{A^3}{E I} - \sum_o^c x y \frac{A^3}{E I})^2 - (\sum_a^o x^2 \frac{A^3}{E I}) (\sum_a^c y^2 \frac{A^3}{E I})} \quad (9)$$

$$H_o = \frac{V_o (\sum_a^o x y \frac{A^3}{E I} - \sum_o^c x y \frac{A^3}{E I}) - \sum_a^o m_L y \frac{A^3}{E I} - \sum_o^c m_R y \frac{A^3}{E I}}{\sum_a^c y^2 \frac{A^3}{E I}} \quad (10)$$

For symmetrical arches, summing over one-half the arch —

$$M_o = \frac{1}{2} \frac{\sum (m_L + m_R) \frac{A^3}{E I}}{\sum y^2 \frac{A^3}{E I}}; \quad H_o = -\frac{1}{2} \frac{\sum (m_L + m_R) y \frac{A^3}{E I}}{\sum y^2 \frac{A^3}{E I}}; \quad V_o = \frac{1}{2} \frac{\sum (m_L - m_R) x \frac{A^3}{E I}}{\sum x^2 \frac{A^3}{E I}}$$

See "Reinforced Concrete Design," Sutherland and Clifford, for a complete illustration of a symmetrical arch solved by this method, although the derivation of equations is by the Method of Least Work.

Temperature Stresses

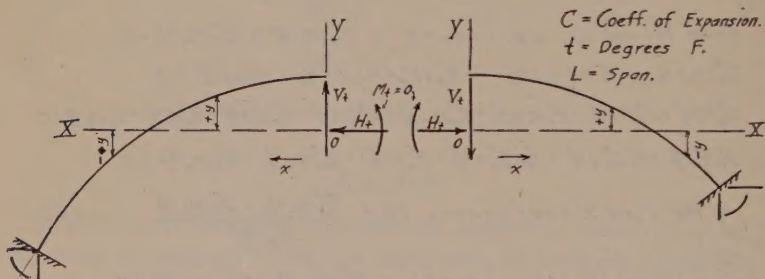


FIG. 5

If the arch be unrestrained, the increase in span length for a rise in temperature of t degrees would be $C \cdot t \cdot L$. This equals Δx .

$$\text{As } m_L \text{ and } m_R \text{ are zero, } M_L = M_t - H_t \cdot y + V_t \cdot x$$

$$M_R = M_t - H_t \cdot y - V_t \cdot x$$

$$1) \Delta \phi = 0 \therefore \Delta \phi_L + \Delta \phi_R = 0 \therefore \sum_a^o M_L \frac{A^3}{E I} + \sum_o^c M_R \frac{A^3}{E I} = 0, \text{ as before.}$$

$$\sum_a^o (M_t - H_t \cdot y + V_t \cdot x) \frac{A^3}{E I} + \sum_o^c (M_t - H_t \cdot y - V_t \cdot x) \frac{A^3}{E I} = 0$$

$$M_t \sum_a^o \frac{A^3}{E I} - H_t \sum_a^o y \frac{A^3}{E I} + V_t \sum_a^o x \frac{A^3}{E I} + V_t (\sum_a^o x \frac{A^3}{E I} - \sum_o^c x \frac{A^3}{E I}) = 0 \therefore M_t = 0 \quad (11)$$

$$2) \Delta x = 0 \therefore \Delta x_L + \Delta x_R + C \cdot t \cdot L = 0 \therefore \sum_a^o M_L y \frac{A^3}{E I} + \sum_o^c M_R y \frac{A^3}{E I} = -C \cdot t \cdot L$$

$$\sum_a^o (M_t - H_t \cdot y + V_t \cdot x) y \frac{A^3}{E I} + \sum_o^c (M_t - H_t \cdot y - V_t \cdot x) y \frac{A^3}{E I} = -C \cdot t \cdot L$$

$$-H_t \sum_a^o y^2 \frac{A^3}{E I} + V_t \sum_a^o x y \frac{A^3}{E I} - V_t \sum_o^c x y \frac{A^3}{E I} = -C \cdot t \cdot L \quad (12)$$

$$\begin{aligned}
 3) \quad \Delta y = 0 \quad \therefore \quad \Delta y_L - \Delta y_R = 0 \quad \therefore \quad \sum_a^o M_L \cdot x \frac{\Delta^2}{E I} - \sum_b^c M_R \cdot x \frac{\Delta^2}{E I} = 0 \\
 \sum_a^o (M_t - H_t \cdot y + V_t \cdot x) x \frac{\Delta^2}{E I} - \sum_b^c (M_t - H_t \cdot y - V_t \cdot x) x \frac{\Delta^2}{E I} = 0 \\
 -H_t \sum_a^o x y \frac{\Delta^2}{E I} + H_t \sum_b^c x y \frac{\Delta^2}{E I} + V_t \sum_a^c x^2 \frac{\Delta^2}{E I} = 0
 \end{aligned} \tag{13}$$

Solving,

$$V_t = \frac{C \cdot t \cdot L \cdot E (\sum_a^o x y \frac{\Delta^2}{E I} - \sum_b^c x y \frac{\Delta^2}{E I})}{\sum_a^c x^2 \frac{\Delta^2}{E I} \cdot \sum_b^c y^2 \frac{\Delta^2}{E I} - (\sum_a^o x y \frac{\Delta^2}{E I} - \sum_b^c x y \frac{\Delta^2}{E I})^2} \tag{14}$$

$$H_t = \frac{C \cdot t \cdot L \cdot E \sum_a^c x^2 \frac{\Delta^2}{E I}}{\sum_a^c x^2 \frac{\Delta^2}{E I} \cdot \sum_b^c y^2 \frac{\Delta^2}{E I} - (\sum_a^o x y \frac{\Delta^2}{E I} - \sum_b^c x y \frac{\Delta^2}{E I})^2} \tag{15}$$

For the symmetrical case, $M_t = 0$; $V_t = 0$; $H_t = \frac{C \cdot t \cdot L \cdot E}{\sum_a^c y^2 \frac{\Delta^2}{E I}}$

Stresses Due to Rib Shortening

Rib shortening due to thrust has the effect on the arch rib of a fall in temperature. If f_c is the average stress in the concrete (equal approximately to the thrust as determined by (10) divided by the cross-sectional area at the crown), it develops from $E = \text{stress/strain}$ that the arch would be shortened an amount $f_c L / E$. This equals $C \cdot t \cdot L$ in (12). Therefore, if $-f_c L / E$ be substituted for $C \cdot t \cdot L$ in (14) and (15), the required values of H and V may be found. (16)

Stresses Due to Shrinkage

In the same manner we may substitute $-C_s \cdot L$ for $C \cdot t \cdot L$, where C_s is the coefficient of shrinkage of the concrete. (17)

Note. — Equations (14), (15) and (17) neglect rib shortening due to the thrust resulting from temperature and shrinkage. To correct for this item, use (16) with values of H as determined from (15) and (17).

Properties of the Section.											
Given					Derived						
1	2	3	4	5	6	7	8	9	10	11	12
No.	ΔS ft.	d ft.	x	y	$\frac{\Delta S}{I}$	$x \cdot \frac{\Delta S}{I}$	$y \cdot \frac{\Delta S}{I}$	$x^2 \frac{\Delta S}{I}$	$y^2 \frac{\Delta S}{I}$	$x \cdot y \frac{\Delta S}{I}$	$x \cdot y \frac{\Delta S}{I}$
10	5.68	3.34	33.94	-15.52	0.150	5.09	-2.33	173.	36.2	-79.0	
9	5.23	3.04	29.94	-11.92	0.187	5.60	-2.23	168.	26.6	-66.8	
8	5.00	2.79	25.94	-8.72	0.231	6.00	-2.01	156.	17.5	-52.3	
7	4.78	2.54	21.94	-5.92	0.291	6.39	-1.72	140.5	10.2	-37.8	
6	4.55	2.33	17.94	-3.52	0.361	6.48	-1.27	116.5	4.5	-22.8	
5	4.45	2.16	13.94	-1.52	0.443	6.18	-0.67	86.	1.0	-9.4	
4	4.26	2.00	9.94	+0.08	0.533	5.30	+0.04	53.	0.0	+0.4	
3	4.17	1.83	5.94	+1.28	0.684	4.06	+0.88	24.	1.0	+5.2	
2	4.05	1.67	1.94	+2.08	0.874	1.70	+1.82	3.	3.8	+3.5	
1	4.01	1.54	2.06	+2.48	1.102	2.27	+2.73	5.	6.8		+5.6
1'	4.01	1.54	6.06	+2.48	1.102	6.67	+2.73	40.5	6.8		+16.6
2'	4.05	1.67	10.06	+2.08	0.874	8.80	+1.82	88.5	3.8		+18.3
3'	4.17	1.83	14.06	+1.28	0.684	9.63	+0.88	135.	1.0		+12.3
4'	4.26	2.00	18.06	+0.08	0.533	9.63	+0.04	174	0.0		+0.8
5'	4.45	2.16	22.06	-1.52	0.443	9.80	-0.67	216.	1.0		-14.9
Σ					8.492		0.00	1579	120.2	-259	+38.7

	13	14	15	16	17	18	19	20
No.	m_L	m_R	$m_L \frac{\Delta S}{I}$	$m_R \frac{\Delta S}{I}$	$m_L y \frac{\Delta S}{I}$	$m_R y \frac{\Delta S}{I}$	$m_L x \frac{\Delta S}{I}$	$m_R x \frac{\Delta S}{I}$
10	49.00		7.35		-114.2		250.	
9	39.00		7.30		-87.0		218.	
8	29.00		6.70		-58.3		174.	
7	21.00		6.11		-36.1		134.	
6	14.97		5.41		-19.0		97.	
5	8.97		3.97		-6.0		55.4	
4	4.97		2.65		+0.2		26.3	
3	2.97		2.03		+2.6		12.0	
2	0.97		0.85		+1.8		1.6	
1		1.03		1.14		+2.8		2.3
1'		3.03		3.34		+8.3		20.2
2'		5.03		4.40		+9.2		44.3
3'		9.03		6.18		+8.0		87.0
4'		15.03		8.02		+0.6		145.0
5'		21.03		9.31		-14.1		206.0
Σ			42.37	32.39	-316.	+14.8	968.3	504.8

Illustrative Problem (Solution of Equations)

Substituting the results of the tabulated computations into (6), (7) and (8), term by term —

$$(6), \quad M_0 = \frac{42.37 + 32.39}{8.492} \quad \therefore \quad M_0 = 8.8'*$$

$$(7), \quad -120.2H_0 - 259V_0 - 38.7V_0 - (-316) - 14.8 = 0$$

$$(8), \quad -(-259)H_0 + 38.7H_0 + 1579V_0 - 968.3 + 504.8 = 0$$

Combining terms in (7) and (8), and solving, $V_0 = -0.334*$
 $H_0 = +3.33*$

Temperature

Substituting into (14) and (15), with $C=0.000\ 006$; $t=50^{\circ}\ F$.

$$(12), \quad V_t = \frac{0.000\ 006 \times 50^{\circ} \times 60' \times 288,000,000 (-259 - 38.7)}{1579 \times 120.2 - (-259 - 38.7)^2} = -15,250''$$

$$(13), \quad H_t = \frac{0.000\ 006 \times 50^{\circ} \times 60' \times 288,000,000 \times 1579}{\text{Previous denom.} = 101,400} = +80,900'$$

In practice, unit loads may be placed at regular intervals along the arch, and H_o , V_o and M_o found for each load. From these quantities influence lines may be drawn from which maximum values of shears, thrusts and moments may be found. Likewise, influence lines may be drawn for the similar quantities at the springing lines and at other points in the arch.

Note. — An influence line shows the variation in moment, thrust, shear, etc., at a selected point in the arch due to a unit load moving over the structure. The ordinates are plotted under the position of the unit load, but the effect is at the selected point.

UNSYMMETRICAL TWO-HINGED ARCH

The two-hinged arch is indeterminate in the first degree. Four equations, three of which are supplied by statics, are necessary for the determination of the reactions. The fourth is given by Equation (2):

$$\Delta x = 0$$

$$\text{i.e.} \quad - \int_A^B M_y \frac{ds}{EI} = 0.$$

As the pins are fixed, it might seem that either $\Delta x = 0$ or $\Delta y = 0$ might be used. But if the deformed arch axis be made to coincide at one end with the unstressed axis, it will be noted that the vertical displacement of the free end is greater than the horizontal displacement of the free end. Thus the horizontal displacement is nearer to zero, and may be taken as equal to zero without introducing appreciable error.

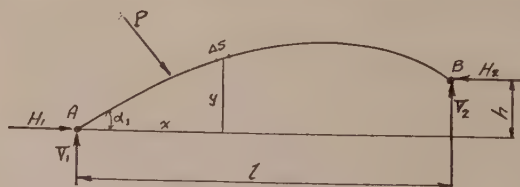


FIG. 7

H and V are the components of reactions, as shown.

The origin is taken at the lower pin to make all y values positive.

m_L = moment about the center of any segment due to applied loads and weight of arch ring acting between the point under consideration and the origin at A .

K = moment of the same loads about the pin B = a constant.

$$\Delta x = 0; -\int_a^b M y \frac{\Delta s}{EI} = 0; \text{ From statics, } M_x = V_r \cdot x - m_L - H_r \cdot y.$$

$$\text{i.e., } -\int_a^b (V_r \cdot x - m_L - H_r \cdot y) y \frac{\Delta s}{EI} = 0; \text{ and } -\int_a^b V_r \cdot x y \frac{\Delta s}{EI} + \int_a^b m_L y \frac{\Delta s}{EI} + \int_a^b H_r \cdot y^2 \frac{\Delta s}{EI} = 0.$$

$$\text{Moments at B, } V_r \cdot l - H_r \cdot h - K = 0 \quad \therefore V_r = H_r \cdot \frac{h}{l} + \frac{K}{l}$$

$$\text{i.e., } -\int_a^b H_r \cdot \frac{h}{l} x y \frac{\Delta s}{EI} - \int_a^b \frac{K}{l} x y \frac{\Delta s}{EI} + \int_a^b m_L y \frac{\Delta s}{EI} + \int_a^b H_r \cdot y^2 \frac{\Delta s}{EI} = 0.$$

$$H_r = \frac{\frac{K}{l} \int_a^b x y \frac{\Delta s}{EI} - \int_a^b m_L y \frac{\Delta s}{EI}}{\int_a^b y^2 \frac{\Delta s}{EI} - \frac{h}{l} \int_a^b x y \frac{\Delta s}{EI}} \quad (18)$$

Equation (18) neglects rib shortening due to thrust and the effect of temperature. The complete expression, in which rib shortening due to thrust is approximate, follows:

$$H_r = \frac{\frac{K}{l} \int_a^b x y \frac{\Delta s}{EI} - \int_a^b m_L y \frac{\Delta s}{EI} + c \cdot t \cdot l \cdot E}{\int_a^b y^2 \frac{\Delta s}{EI} - \frac{h}{l} \int_a^b x y \frac{\Delta s}{EI} + \frac{L \cdot \cos \alpha_1}{A_o}} \quad (19)$$

in which c = coefficient of expansion; t = rise in temperature in degrees F.; l = span; L = length of arch axis; A_o = cross-sectional area of the arch at the crown; and α_1 = inclination of arch axis at springing line.

Note. — If the supports are on the same level, and M' is the moment due to loads and vertical reactions only —

$$H = \frac{\int_a^b M' y \frac{\Delta s}{EI} + c \cdot t \cdot l \cdot E}{\int_a^b y^2 \frac{\Delta s}{EI} + \frac{L \cdot \cos \alpha_1}{A_o}} \quad (20)$$

Procedure

The arch is divided into segments, as in the case of the hingeless arch, the properties of each segment tabulated, and the summations found. Then H_1 is found from the given equation, whereupon the remaining components of reactions and the moment at any point on the arch may be found from statics.

Discussion

O. G. JULIAN:* Professor Weaver has given us a most clear, comprehensive and concise analysis of a right arch.

He states that "he assumes contrary to fact that E and I are known" At the present state of the art it is doubtful if the true values of E and I can be even approximately known, and it is a moot question whether EI *per se* has any place in the exact analysis of important concrete structures.

According to Bulletin No. 185 (page 10) of the Engineering Experiment Station, University of Illinois, even for short-time and comparative low loads concrete shows an appreciable amount of inelastic, *i.e.*, plastic, deformation. This conclusion is borne out by other tests. (See "Principles of Reinforced Concrete Construction," by Turneaure and Maurer, 4th edition, Fig. 5, page 21.) Accordingly, it might appear that the so-called "elastic theory" as applied to concrete structures is based on false premises.

At the risk of appearing pedantic, I am going to get down to elements.

By changing mass to area, structural engineers make the fundamental definition of moment of inertia read as follows (notation as per Professor Weaver's Fig. 1):

$$I = \int z^2 dA \quad (1)$$

By statics —

$$M = \int z f dA \quad (2)$$

and *if* the fiber stress (f) varies directly as the strain, *i.e.*, $f = cz$, it immediately follows that —

$$M = c \int z^2 dA = cI = \frac{fI}{z} = \frac{eEI^\dagger}{z} \quad (3)$$

Also, as Professor Weaver has shown —

$$\frac{d\phi}{ds} = \frac{M}{EI} \quad (4)$$

For composite members, the I 's and A 's in the above formulas are, of course, those of the transformed section.

* Structural Engineer with Jackson & Moreland, Consulting Engineers, 31 St. James Avenue, Boston, Mass.

† c = a constant, and e = strain.

In both (3) and (4) E , by hypothesis, does not vary with the intensity of stress, but for a given material is constant over the cross section. For materials which approach perfect elasticity (e.g., steel worked within the proportional limit), this theory gives quite accurate results. However, concrete does not belong to this class of materials. It is partly plastic and partly elastic, and has the characteristics of a viscous fluid.* If E is defined as the strain derivative of stress, E for concrete is not equal to $\frac{f}{e}$, and f is not equal to cz . For a given concrete E varies inversely as functions of the intensity of stress and the duration of the loading and directly as a function of the age of the concrete.

It appears reasonable and it is borne out by tests, that —

$$f = CE_0 e^m \quad (5)$$

in which C and m are constants to be determined from tests of the given concrete *at the age and for the duration of loading being considered*, e is strain and E_0 is the initial value of E ; i.e., the slope of the stress-strain curve at the origin and for short-time loading.

Then, by taking the strain derivative of stress —

$$\frac{df}{de} = E = CE_0 m e^{m-1} \quad (6)$$

which, if m equals unity, *but not otherwise*, reduces to —

$$E = E_0 = \frac{f}{e} \quad (7)$$

For 3,000 pounds per square inch concrete, loaded when 28 days old, E_0 = approximately 4.9×10^6 pounds per square inch while with $f_c = 750$ pounds per square inch, E_{750} = approximately 3.6×10^6 pounds per square inch.†

There is some increase in E as the concrete ages. However, this is small as compared to the effect of shrinkage and viscous strain. The relationship between strain (e) and time is startling. It is well illustrated by Fig. A which is adapted from "Plain and Reinforced Concrete Arches,"

* For definitions of plasticity, elasticity and viscosity, as here used, see Maxwell's "Theory of Heat," p. 295. Also Transactions, American Society of Civil Engineers, Vol. 95 (1931), p. 689.

† From "Principals of Reinforced Concrete Construction," by Turneaure and Maurer, 4th edition, p. 19.

by Charles S. Whitney, American Concrete Institute Journal, March, 1932, page 485.*

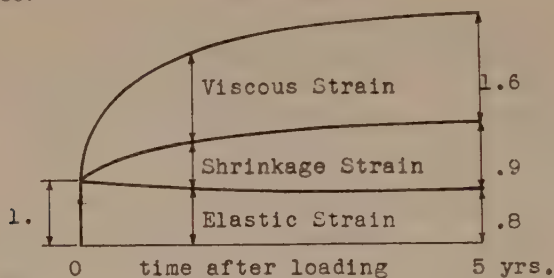


FIG. A.— VARIATION IN STRAIN WITH TIME; LOAD CONSTANT

Age of concrete at origin approximately three months. Values shown for strains are approximate relative values.

Adapted from Fig. 3, "Plain and Reinforced Concrete Arches," by Charles C. Whitney, American Concrete Institute Journal, March, 1932, p. 485.

Based on equations somewhat similar to (5), a beautiful analysis applicable to symmetrical, plain concrete arches has been made by Prof. Lorenz G. Staub.[†] His analysis involves neither E nor I *per se*.

Aside from the involved expressions to which they lead, the principal trouble with such equations as (5) and (6), and analyses based on them, is that, although we have been using reinforced concrete for over eighty years, and have been building masonry arches for a great deal longer period, we have but the faintest notions as to the values of the constants in such equations. Until we have definite data as to the values of these constants, we cannot compute, even approximately, the absolute values of the strains and stresses in concrete structures.

However, this does not mean that we cannot approximate the moment distribution and the magnitude of the thrust. For well made concrete, and for the range of ordinary conservative working stresses (less than one-third of the ultimate strength), the variation of E_c with

* See also "Flow of Concrete Under Sustained Compressive Stress," by Prof. Raymond E. Davis, American Concrete Institute Proceedings, 1928, p. 303 *et seq.*; "Flow of Concrete under the Action of Sustained Loads," by Raymond E. Davis and Harmer E. Davis, American Concrete Institute Journal, March, 1931, p. 837 *et seq.*; and discussions in American Concrete Institute Journals for June and December, 1931; "Creep of Concrete Under Load," by W. H. Glanville and F. G. Thomas, in "The Structural Engineer," London, Eng., February, 1933, p. 54 *et seq.*

[†] See Transactions, American Society of Civil Engineers, Vol. 95, 1931, p. 613 *et seq.*

stress is not enormous, and may, with some degree of accuracy, be regarded as constant. Then, as Professor Weaver has shown, E *per se* cancels from the equations for M and H ; but it appears disguised as n in the expression for I of the transformed section. In this form, assuming that there is no great variation in the percentage of reinforcement, it affects the value of all $\frac{\Delta s}{I}$'s about equally. Since $\frac{\Delta s}{I}$ appears in the same degree in all terms of Professor Weaver's equations (6), (16) and (17), it may be concluded that small changes in I , brought about by variations in E_c , do not appreciably affect the values of M , V and H . However, it will be noted that this is not true of equations (12) and (13) for V_t and H_t , and hence these later mentioned equations may give results in which the errors are quite material.

The following quotations from Professor Staub's conclusions are of a reassuring nature and may be in order:

The fact that the ratio of stress to elastic strain does not follow Hooke's law for concrete usually causes but little difference in moment distribution from that computed by the ordinary elastic theory. For rib shortening and temperature changes the difference may be greatly on the side of safety.

. . . When no external disturbances occur, plastic flow will cause progressive changes in the deflections, . . . but will cause little or no change in the moment distribution. . . .*

A little consideration indicates that in so far as simple flexure is concerned, the elastic theory is just as reliable as the commonly accepted beam formula —

$$M = \frac{If}{z} \quad (3)$$

The foundation of the elastic theory is —

$$\frac{d\phi}{ds} = \frac{M}{EI} \quad (4)$$

and the derivation of this and the beam formula are based on the same premises.

Considering combined bending and direct stress, since ordinarily the entire section (or practically the entire section) is in compression —

$$f = \frac{P}{A} + \frac{Mz}{I} \quad (8)$$

* *Loc. cit.*, p. 677.

applies.* The assumptions made in the derivation of this formula and the elastic theory are identical.

It is to be emphasized that in computing stresses from thrusts and moments, the actual E pertaining to the age of the concrete, the magnitude of fiber stress, and the *duration of the loading being considered* should be employed. E for loadings of long duration such as dead load may be of the order of 7.5×10^5 (n =approximately 40),† while E_o , which is more nearly applicable for live load and temperature stresses, may be about 4.9×10^6 (n =approximately 6).

For a constant loading, while shrinkage and viscous strain decrease the concrete stress (f_c) somewhat, a concomitant and very much greater increase (perhaps well over 100 per cent) occurs in the steel stress (f_s).‡ For constant strain, the steel stress remains constant, while the concrete stress will materially decrease with time.

It should be borne in mind that any abutment or pier will yield, vertically, transversely and as regards rotation. The deflections and resultant stresses from such yielding as may be anticipated should be given careful consideration.

Definite data regarding the relations between strain, stress and time for given water cement ratios, mixes, methods of curing, etc., are not readily available. In the absence of such data the guesses we are forced to make regarding the elastic behavior of concrete render precise calculations of the stresses and deflections a waste of time, but this in no way renders the elastic theory useless (as has been claimed by some). Calculations based on it may be reasonably accurate without being precise.

* Since the depth of the cross section is small in comparison to the radius of the arch axis, the more complicated formulas pertaining to curved beams reduce, practically, to the forms given. For an excellent discussion regarding planar distribution of stress in curved beams, and the applicability of equations (3) and (8) to such members, see Chapter XX, "Strength of Materials," by George Fillmore Swain; also "Strength of Materials," by S. Timoshenko, Pt. II, p. 423 *et seq.*

† See also "Principles of Reinforced Concrete Construction," 4th edition by Turneure and Maurer, p. 349 *et seq.*

‡ See numerical examples given in "Plain and Reinforced Concrete Arches," by Charles S. Whitney, American Concrete Institute Journal, March, 1932, p. 485 *et seq.*; "Stresses in Reinforced Concrete Due to Volume Changes," by C. P. Vetter, Proceedings American Society of Civil Engineers, February, 1932, p. 197 *et seq.*, and discussion on same (giving numerical examples) by Ralph E. Spaulding, Proceedings, American Society of Civil Engineers, September, 1932, p. 1243 *et seq.*; "Second Progress Report on Column Tests at the University of Illinois," by F. E. Richart and G. C. Staehle, American Concrete Institute Journal, March, 1931, p. 761 *et seq.*; "Second Progress Report on Column Tests Made at Lehigh University," by W. A. Slater and Inge Lyse, American Concrete Institute Journal, March, 1931, p. 791 *et seq.*; Discussion of the two latter papers in American Concrete Institute Journal for June, 1931, p. 1297 *et seq.*; "Tests Provide New Data on Concrete-Column Design," Engineering News-Record, May 26, 1932. This article gives in condensed form the results of tests at the University of Illinois and Lehigh University. An increase in f_s from 6,000 pounds per square inch to 37,000 pounds per square inch is cited.

OF GENERAL INTEREST

PROCEEDINGS OF THE SOCIETY

MINUTES OF MEETINGS

Boston Society of Civil Engineers

APRIL 18, 1933. — A regular meeting of the Boston Society of Civil Engineers was held this evening at the Engineers Club, and was called to order by the President, Arthur W. Dean. There were about 110 members and guests present. Previous to the meeting 86 persons attended the buffet supper.

The President announced the death of James Edgar Borden, who had been a member since May 15, 1915, on January 20, 1933.

The Secretary announced that the following had been elected to membership:

Grade of Member: Henry A. Mohr, Almon Charlie David,* Harold W. Kelly.*

The President introduced Mr. Albert E. Kleinert, Jr., Assistant Structural Engineer, Massachusetts Department of Public Works, who presented a paper on the "Design and Construction of the French King Bridge."

The French King Bridge, constructed in a location unique for scenic grandeur, and interesting historically, spans the Connecticut River north of Millers Falls, on the new cut-off constructed by the Massachusetts Department of Public Works during the summer of 1932 on the Mohawk Trail route between the towns of Erving and Greenfield. This bridge has an overall length of 890 feet, and the roadway surface is 135 feet above the river. The design is of unusual type, and the construction methods and procedure involved were modern and expen-

ditious. The closing of the two halves of the bridge at the center of the span and the jacking of the shoes are particularly interesting.

The paper was illustrated by lantern slides and moving pictures.

The meeting adjourned at 9.10 P.M.

EVERETT N. HUTCHINS, *Secretary*.

Designers Section

APRIL 12, 1933. — A meeting of the Designers Section of the Boston Society of Civil Engineers was held on Wednesday, April 12, 1933, and was called to order at 6.15 P.M., by the Chairman, Prof. Frederic N. Weaver, who introduced Dr. Raymond V. Phelan, economist, of Tufts College. Dr. Phelan discussed various causes of our present economic dilemma, some proposed solutions, and also told us how and when in his opinion we would pull out of this depression. There was a lively discussion following his paper.

The meeting adjourned at 8.30 P.M. There were 20 members and guests present.

HENRY BRASK, *Clerk*.

APPLICATIONS FOR MEMBERSHIP

[June 20, 1933]

THE By-Laws provide that the Board of Government shall consider applications for membership with reference to the eligibility of each candidate for admission and shall determine the proper grade of membership to which he is entitled.

* Transferred from Grade of Junior.

The Board must depend largely upon the members of the Society for the information which will enable it to arrive at a just conclusion. Every member is therefore urged to communicate promptly any facts in relation to the personal character or professional reputation and experience of the candidates which will assist the Board in its consideration. Communications relating to applicants are considered by the Board as strictly confidential.

The fact that applicants give the names of certain members as reference does not necessarily mean that such members endorse the candidate.

The Board of Government will not consider applications until the expiration of fifteen (15) days from the date given.

For Admission

BROGLINO, THOMAS H., Boston, Mass. (Age 29, b. Boston, Mass.) Graduate of Berkeley Preparatory School, and three years (1924-27) Massachusetts Institute of Technology. Experience: three years with Gibbs & Hill on electrification work, Pennsylvania Railroad in New York; Otis Elevator Company, two months; surveying for six months for J. Pearson, Boston; civil engineer, with Schuyler & Clapp, seven and one-half months; construction computation, one and one-half years, with Frank Barr. At present employed at Hotel Statler. Refers to *C. B. Breed, L. M. Hersum, J. W. Howard, W. H. Lawrence, C. M. Spofford.*

HASELTINE, STEPHEN, JR., Stoneham, Mass. (Age 23, b. Somerville, Mass.) Attended Northeastern University four years, graduating in 1931, with degree of B.C.E. During co-operative periods was employed by R. H. Barnes of Waltham as rodman; two years as transitman, chief and labor foreman in the Stoneham Public Works Department; resident engineer two months for Morse & Dickinson on a road construction project in Hamilton, Mass.; foreman and assistant superintendent of paving construction on bituminous macadam pavements for A. N. Nickerson; at present is serving as a sales engineer for the Holland Furnace Company of Michigan. Refers to *H. B. Alvord, J. G. Carlson, K. H. Gibson, J. H. Lowe.*

MULLER, ARTHUR W., Roslindale, Mass. (Age 20, b. Roslindale, Mass.) After graduating from high school in 1930, started a course on building of structures at Franklin Union, and has now completed three years of the course. Has had very little experience owing to the depression. Refers to *Robinson Abbott, S. L. Conners, L. M. Hersum, E. N. Hutchins.*

POWER, JOSEPH A., Boston, Mass. (Age 34, b. Boston, Mass.) After graduating from high school entered the employ of Bay State Street Railway Company as rodman, June, 1916; in June, 1917, accepted a position with the New York, New Haven & Hartford Railroad in the engineering department, in all capacities from rodman to transitman, chief of party; has had charge of surveys and laying out work in the field, estimating and plan making; is now inspector for this same company. Refers to *C. P. Foyer, N. W. Littlefield, F. P. Nickerson.*

ADDITIONS

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RICHARD L. DUNNING, 247 Kent Street, Brookline, Mass.
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 CLARENCE BENSON, 104 Bates Street, Roslindale, Mass.
 JOSEPH BOCON, JR., 65 Sewall Avenue, Brookline, Mass.

Junior

PAUL A. BEIGBEDER, Temporarily, Lone Star Gardens, Thomasville, Ga.

DEATH

J. EDGAR BORDEN . . . Jan. 20, 1933

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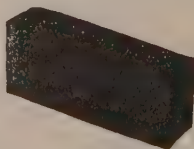
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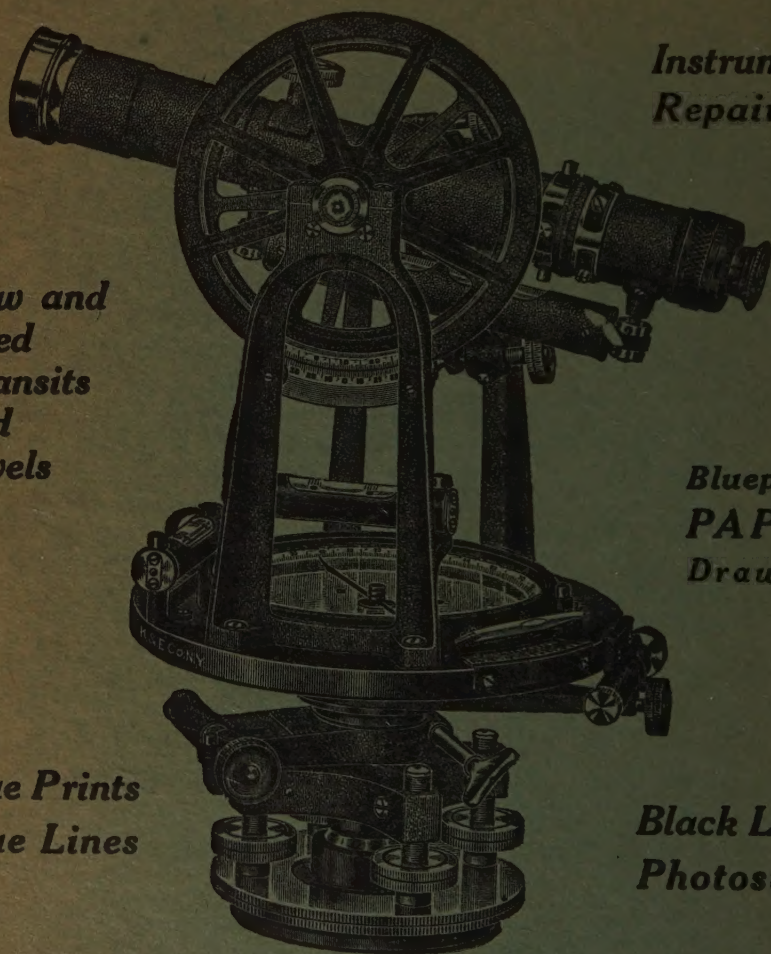
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